LITERATURE CITED

1. L. I. Sedov, Similarity and Dimensional Methods in Mechanics [in Russian], Nauka, Moscow (1972).
2. N. N. Kochina and N. S. Mel'nikova, "A strong point explosion in a compressible medium," Prikl. Mat. Mekh., 22, No. 1 (1958).
3. G. M. Lyakhov, Principles of the Dynamics of Explosion Waves in Soils and Rocks [in Russian], Nedra, Moscow (1974).
4. G. M. Lyakhov, V. N. Okhitin, and A. G. Chistov, "Shock waves in soils and in water near the point of an explosion," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1972).
5. V. A. Krivtsov, "The equation of the compressibility of water-saturated sand and the limits of its applicability," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1974).
6. G. M. Lyakhov and V. N. Okhitin, "Spherical explosion waves in multicomponent media," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1974).

DETERMINATION OF EJECTION EXTRACTION WITH THE EXPLOSION OF AN UNDERGROUND FUSE-TYPE CHARGE IN A TWO-LAYER MEDIUM

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In [1], in a pulsed-hydrodynamic statement, an investigation was made of the problem of determining the ejection extraction with the explosion of a fuse-type discharge in a two-layer medium. This problem is solved below with more general assumptions.

Let the ground consist of two layers of identical density, differing in the values of the critical velocity. The upper layer with a thickness H is characterized by the critical velocity v_1, while the lower layer, of unbounded thickness, has the critical velocity v_2. At a depth h from the surface of the ground, there is a fuse-type charge, modeled in the vertical plane by a source with a power of 2q. It is required to determine the limit of the ejection extraction, taking account of its lines of flow, and taking $v = v_1$ behind it in the upper layer and $v = v_2$ in the lower layer, where v is the value of the velocity. We note that, in distinction from a solid–liquid model of an explosion (see, for example, [2, 3]), in the present work, as in [1], the condition $v > v_0$ (v_0 is the critical velocity) is not imposed in the region of the motion. Only such ejection schemes are considered in which the point of branching of the boundary of the ejection extraction lies below the line of separation of the layers. Depending on the ratio of the critical velocities v_1 and v_2, two variants are studied.

Variant 1. Let $v_1 < v_2$. The corresponding scheme of the ejection extraction is illustrated in Fig. 1 (by virtue of the symmetry with respect to the y axis, only the right-hand half of the ejection extraction is shown; this region is denoted by G_Z and its boundary ABMNRCDA, by Γ_Z). We note that the condition $|y_0| \geq H_0$ (y_0 is the value of y at the point B) is clearly satisfied if $h \geq H$. The starting parameters of the problem are q, h, H, v_1, and v_2.

We introduce dimensionless variables by the relationships

$$z^* = z/H, \quad w^* = w/q, \quad v^* = vH/q, \tag{1}$$

where $z = x + iy$ is the physical plane; $w(z) = \varphi + i\psi$ is the complex flow potential. The solution will then depend only on three parameters:

$$h^* = h/H, \quad v_1^* = v_1H/q, \quad v_2^* = v_2H/q,$$

since $H^* = 1$, $q^* = 1$. In what follows, for simplicity, we shall omit the superscript asterisk for the dimensionless variables.

The problem described reduces to the following boundary-value problem: Determine the unknown sections of the boundary Γ_Z of the region G_Z in such a way that the function $w(z) = \varphi(x, y) + i\psi(x, y)$, analytical in G_Z and continuous in $\overline{G}_Z$ (except for the point A), will satisfy the following conditions at Γ_Z:

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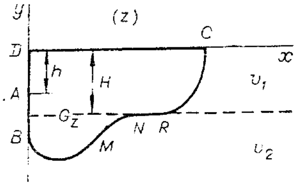

Fig. 1

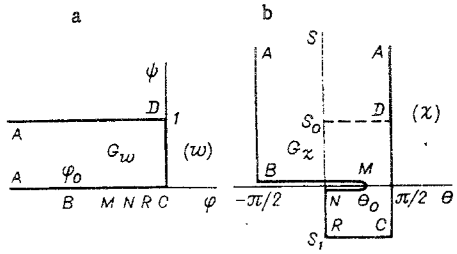

Fig. 2

$$\varphi = 0 \text{ at } DC, \ \psi = 0 \text{ at } ABMNRC, \ \psi = 1 \text{ at } AD; \tag{2}$$

$$\partial\varphi/\partial s = v_1 \text{ at } RC, \ \partial\varphi/\partial s = v_2 \text{ at } BMN, \tag{3}$$

where s is the arc abscissa of a point of the boundary Γ_Z. At the point A, the function w(z) must have a logarithmic singularity.

In the plane w, taking account of the condition (2), the solution will correspond to the semiaxis G_W (Fig. 2a); here the parameter φ_0 (the value of φ at the point B) is not determined. We introduce the function

$$\chi = i \ln\left(\frac{1}{v_2}\frac{dw}{dz}\right) = \theta + iS, \tag{4}$$

where θ is the angle of inclination of the velocity vector to the x axis; $S = \ln(v/v_2)$. In the plane χ, taking account of condition (3), the solution will correspond to the region G_χ (Fig. 2b); the corresponding points in the different planes are designated by the same letters. The parameters S_0 (the value of S at the point D) and θ_0 (the value of θ at the point M) are not determined; $S_1 = \ln(v_1/v_2)$.

We note that, by the postulation of the existence of a point of inflection at the boundary of the ejection, BN is permissible in the region G_Z near the section MN with $v < v_2$. In fact, the presence of a point of inflection M leads to the appearance of a section MN of the boundary Γ_2 (we designate it by γ_Z), concave inside the region, i.e., such that $\partial\theta/\partial s \le 0$. Then, examining the function $\ln(dw/dz)(z) = \ln v - i\theta$, analytical near γ_Z, in accordance with the Cauchy–Riemann conditions, $\partial \ln v/\partial n = -\partial(-\theta)/\partial s$ (n is the internal normal to γ_Z) we obtain $\partial v/\partial n \le 0$. Consequently, in G_Z there is a region adjacent to γ_Z in which $v < v_2$. What has been said can be seen from Fig. 2b: At any given point near the section MN of the region G_χ we have $S < 0$, i.e., $v < v_2$.

Taking the half-plane $\operatorname{Im}\zeta > 0$ ($\zeta = \xi + i\eta$) as a canonical region, we map it conformally on the region G_W by the function

$$w = i\left(\frac{1}{2} + \frac{1}{\pi}\arcsin\zeta\right); \tag{5}$$

here the points A, B, M, N, R, C, and D correspond to the points $\pm\infty$, b, m, n, r − 1, 1 of the ξ axis; where $-\infty < b < m \le n < r \le -1$. We then map this half-plane on the region G_χ by the Christoffel–Schwarz integral

$$\chi = i\int_b^\zeta \frac{(\tau - m)\,d\tau}{\sqrt{(\tau - b)(\tau - n)(\tau - r)(\tau + 1)}} - \frac{\pi}{2}. \tag{6}$$

We note that the coefficient ahead of the integral is equal to i, as a consequence of the fact that, at the point $\zeta = \infty$, $\operatorname{Re}\chi(\zeta)$ undergoes a discontinuity, equal to π.

Differentiating the function (5), and using relationships (4) and (6), we find

$$z = \frac{1}{\pi v_2}\int_1^\zeta \exp\left[-\int_b^\zeta \frac{(\tau - m)\,d\tau}{\sqrt{(\tau - b)(\tau - n)(\tau - r)(\tau + 1)}}\right]\frac{d\zeta}{\sqrt{1 - \zeta^2}}. \tag{7}$$

The expression obtained for $z(\zeta)$ contains four parameters (m, n, r, b); to seek them, we have the system of nonlinear equations

$$\operatorname{Re}\chi(n) = 0, \ \operatorname{Im}\chi(r) = S_1; \tag{8}$$

$$\operatorname{Im} z(r) = -1, \ \operatorname{Im} z(\infty) = -h. \tag{9}$$

The values of θ_0 and S_0 are determined from the relationships

$$\theta_0 = \operatorname{Re}\chi(m), \quad S_0 = \operatorname{Im}\chi(1). \tag{10}$$

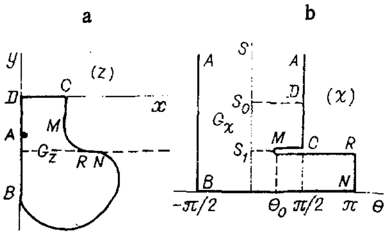

Fig. 3

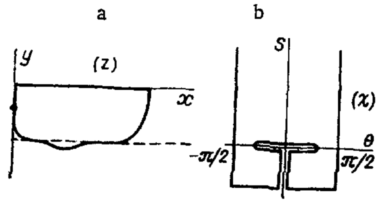

Fig. 4

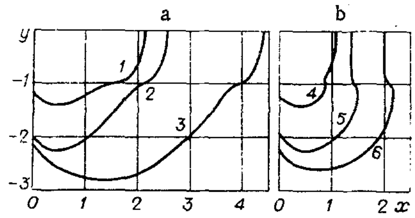

Fig. 5

A partial case of the variant under consideration is the case where $v_1 = 0$. Here, in the plane ζ we will have $r = -1$.

Taking into consideration that the function $\operatorname{Re}\chi(\zeta)$ undergoes a discontinuity equal to $\pi/2$ at the point $\zeta = -1$ and that, at the same point, the function $\operatorname{Im} z(\zeta)$ also undergoes a discontinuity, equal to 1, correspondingly we have

$$-m = 1 + \frac{1}{2}\sqrt{(1+b)(1+n)}, \quad v_2 = \frac{\sqrt{-1-b}-\sqrt{-1-n}}{\sqrt{-1-b}+\sqrt{-1-n}}\sqrt{\frac{2(1+b)(1+n)}{n-b}}. \tag{11}$$

Adding the condition $\operatorname{Im} z(\infty) = -h$ to relationships (11), we will have a system of equations for determining the parameters m, n, and b.

Variant 2. Let $v_1 > v_2$. The scheme of the ejection extraction and the region corresponding to it in the plane χ are shown in Fig. 3a, b. In this case, the region in the plane w has the same form as in the first variant (see Fig. 2a). Mapping of the half-plane $\operatorname{Im}\zeta > 0$ (in the given case $-\infty < b \le n < r < m \le -1$) on the regions G_w and G_χ is effected, respectively, by the functions (5) and (6), and, on the region G_z, by the function (7). For seeking the parameters n, r, m, and b we have the system of equations

$$\operatorname{Re}\chi(n) = \pi, \quad \operatorname{Im}\chi(r) = S_1; \tag{12}$$

$$\operatorname{Im} z(r) = -1, \ \operatorname{Im} z(\infty) = -h. \tag{13}$$

The values of θ_0 and S_0 are determined from the relationships (10).

A partial case of this variant, investigated in [1], is the case where $v_2 = 0$. In the plane ζ, we have $n = b$.

Mapping of the half-plane $\operatorname{Im}\zeta > 0$ on the region G_z is effected in this case by the function

$$z = \frac{1}{\pi v_1}\int_1^\zeta \exp\left[-\int_{-1}^{\zeta}\frac{(\tau-m)\,d\tau}{(\tau-b)\sqrt{(\tau+1)(\tau-r)}}\right]\frac{d\zeta}{\sqrt{(1-\zeta^2)}}.$$

Taking into consideration that, at the point $\zeta = b$, $\operatorname{Re}\chi(\zeta)$, where $\chi = i\ln[(1/v_1)(dw/dz)]$, undergoes a discontinuity equal to $3\pi/2$, we have

$$m = (3/2)\sqrt{(b-r)(b+1)} + b. \tag{14}$$

Adding to the relationship (14) the conditions

$$\operatorname{Im} z(r) = -1, \ \operatorname{Im} z(\infty) = -h,$$

we obtain a system of equations for determining the parameters b, r, and m.

Numerical Calculations. The numerical results were obtained using a semiinverse method, the essence of which consists in the following. Assigning the values of φ_0 (and, consequently, b), S_1, and θ_0, from Eqs. (8) [for the second variant, correspondingly, (12)] and the first of relationships (10), the values of m, n, and r were sought. Then, from the first of Eqs. (9) [correspondingly, (13)], the value of v_1 was

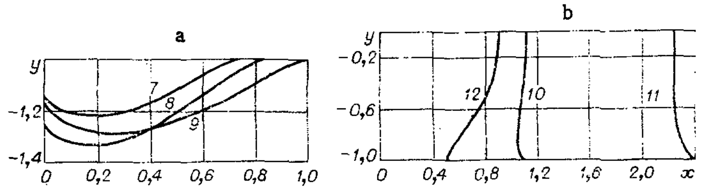

Fig. 6

TABLE 1

Example No. from Fig. 5	1	2	3	4	5	6
v_1	0,16	0,16	0,031	0,63	0,63	0,41
v_2	0,42	0,42	0,079	0,25	0,25	0,16
h	0,91	1,76	0,91	0,81	1,53	1,53
l	0,107	0,047	0,038	0,0057	0,0057	0,0057

TABLE 2

Example No. from Fig. 6	7	8	9	10	11	12
v_1	0	0	0	0,75	0,28	0,75
v_2	1,21	1,21	0,81	0	0	0
h	1	1,14	1	1	1	0,65

determined (and, consequently, from the known value of S_1, the value of v_2), and from the second equation, the value of h. Substituting the values obtained into (7), the unknown sections of the boundary Γ_z were sought.

Since, in a semiinverse statement, the dimensions of the region G_χ are completely unknown, as one of the simplest methods for seeking the parameters m, n, and r, the EHDA method was used [4]. However, the values of the parameters found using this method have a certain error; therefore, their values were refined in the following manner. Eliminating the value of m from Eqs. (8) [correspondingly, (12)], and assuming that the parameter n is determined exactly by the EHDA method, the value of r was sought. Then, from the first of Eqs. (8) [correspondingly, (12)], the value of m was determined, after which the value of θ_0 was refined using the first of relationships (10).

In constructing the region G_χ it was assumed that the following condition holds:

$$\operatorname{Im} z(b) \leqslant -1. \tag{15}$$

If this condition is not satisfied, there is a change in the form of the ejection excavation and, correspondingly, of the region in the plane χ (for the first variant, the form of the above regions is shown in Fig. 4). Therefore, with a solution of the problem in a semiinverse statement, after obtaining the values of the parameters, the satisfaction of condition (15) must be assured.

The above method was used on an M-222 computer to make a series of calculations. The program was set up in the input language of the TA-1M translator. The most interesting results obtained for the cases $S_1 = -3\pi/10$ and $S_1 = 3\pi/10$ are given in Fig. 5a, b.

Examples 1 and 3, 5 and 6 show ejection extractions formed with the explosion of identically arranged charges of different powers. Examples 1 and 2, 4 and 5 correspond to different depths of charges of identical power. The corresponding values of v_1, v_2, h, and the length l of the section RN of the boundary Γ_z for the above six examples are given in Table 1.

Figure 6a, b gives the results of calculations of six examples for the partial cases $v_1 = 0$ and, correspondingly, $v_2 = 0$. The values of v_1, v_2, and h are shown in Table 2. As above, examples 7 and 9, 10 and 11 correspond to identically arranged charges of different powers, and examples 7 and 8, 10 and 12 correspond to depths of charges of identical power.

From the results given, the following conclusions can be drawn.

1. For the case $v_1 < v_2$, the sector is easily followed at the line of separation between the media; with $v_1 > v_2$, this sector is less clearly marked.

2. With a decrease in the strength of the soil, with an identical arrangement of the charge, which, in dimensional variables [taking account of (1)], corresponds to an increase in the power of the charge with exactly the same soil, the dimensions of the ejection excavations increase, both in width and in depth. It must be noted that, in this case, the increase in the width of the excavation takes place at the expense of the lower layer.

3. For identical values of the critical velocities, the dimensions of the ejection extraction increase with an increase in the depth of the charge. Here the increase in the half-width takes place only up to a certain limit.

For the case of a homogeneous soil, characterized by the critical velocity v_0, as follows from the results of [3], this limit as $h \to \infty$ is equal to $2q/v_0$ (the values of h, q, and v_0 are dimensional).

LITERATURE CITED

1. É. B. Polyak and E. N. Sher, "The form of the ejection funnel with the explosion of a fuse-type discharge in a two-layer medium," Zh. Prikl. Mekh. Tekh. Fiz., No. 2, 143-146 (1973).
2. V. M. Kuznetsov, "The form of the ejection funnel with an explosion at the surface of the ground," Zh. Prikl. Mekh. Tekh. Fiz., No. 3, 152-156 (1960).
3. V. M. Kuznetsov and É. B. Polyak, "Pulsed-hydrodynamic schemes with ejection by the explosion of fuse-type charges," Fiz.-Tekh. Probl. Razrab. Polezn. Iskop., No. 4, 32-39 (1973).
4. P. F. Fil'chakov, "Determination of the constants of the Christoffel–Schwarz integral using modeling on electrically conducting paper," Ukr. Mat. Zh., 13, No. 1, 72-79 (1961).

PROPAGATION OF A TWO-DIMENSIONAL PLASTIC WAVE IN A MEDIUM WITH NONLINEAR UNLOADING

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The plane stationary problem about the effect of a moving load on a nonlinearly compressed half-plane is considered.

The case of linear loading and unloading of the medium has been examined in [1, 2]. The solution of the problem obtained in [1] by conformal mapping holds in the case when the propagation velocity a_1 of the unloading strains exceeds the velocity of loading motion. This problem is solved in [2] without constraints on the velocity a_1 by the Mellin integral transform method for the triangular load case.

The influence of nonlinear properties of the half-plane material on the propagation of shock-wave processes therein is studied by the numerical method of characteristics and analytically in this paper.

The computational scheme proposed can be used to determine the parameters of an inhomogeneous medium for different profiles of a given load.

Let a decreasing normal load move monotonically at a constant velocity D exceeding the propagation velocity of the loading–unloading strains of the medium over the surface of a half-plane. The load profile does not change as the wave propagates. The medium filling the half-plane possesses such mechanical properties that the relation between the pressure p and the volume strain ε is nonlinear and irreversible during loading and unloading, where $dp/d\varepsilon > 0$, $d^2p/d\varepsilon^2 > 0$ and the slope of the unloading branch of the $p \sim \varepsilon$ diagram exceeds the slope of the loading branch.

In this case, a shock with the curvilinear surface Σ will be propagated in the half-plane, and the perturbation domain will be bounded by the front Σ and the boundaries of the half-plane. It is assumed that the medium is loaded instantaneously at the front Σ, while unloading occurs in the perturbed domain behind the front. The relationships

$$\rho_0 a = \rho^* (a - v_n^*), \quad \rho_0 a v_n^* = p^*, \quad v_\tau^* = 0 \qquad (a = D \sin\alpha) \tag{1}$$

hold on the surface of strong discontinuity Σ from the mass and momentum conservation conditions. We represent the equation of state of the medium in the form of a polynomial:

$$p^* = \alpha_1 \varepsilon^* + \alpha_2 \varepsilon^{*2}.$$

We have

$$D\frac{\partial u}{\partial \xi} + \frac{1}{\rho}\frac{\partial p}{\partial \xi} = 0, \quad D\frac{\partial v}{\partial \xi} + \frac{1}{\rho}\frac{\partial p}{\partial \eta} = 0,$$
$$D\frac{\partial p}{\partial \xi} + \rho\left(\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \eta}\right) = 0, \quad p = p^* + \beta_1(\varepsilon - \varepsilon^*) + \beta_2(\varepsilon - \varepsilon^*)^2 \tag{2}$$

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